CHAPTER – II

SEQUENCING MODELS

The basic models in scheduling due to Johnson (1957) and owing to Maggu & Das (1977) T.P. Singh (1985, 86, 2005, 2006) are explained one by one which form a basis of scheduling problems dealt in this dissertation. Johnson's (1954) considered the very simple case of n jobs to be processed on two machines A and B, each job requiring the same sequence of operations and no passing allowed. Whichsoever jobs is processed first on machine A must also be processed on machine B and whichsoever job is processed second on machine A also be processed on machine B i.e. the flow shop model.

2.1 Notations, Terminology and Assumptions

NOTATIONS

t_{ij} = Processing time (time required) for job I on machine j.

T = Total elapsed time for processing all the jobs. This includes idle time, if any.

T_{ij} = Idle time on machine j from the end of job (i-1) to the start of job i.

TERMINOLOGY

- Number of Machines: The number of machines refer to number of service facilities through which a job must pass before it is assumed to be completed.
- Processing time: The time required by a job on

each machine.

- Processing Order: It is the order (sequence) in which machines are required for completing the job.
- Idle Time on a Machine It is the time for which a machine does not have a job to process, i.e. idle time from the end of job (i-1) to the start of job i.
- Total Elapsed Time: It is the time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of machines.
- No Passing Rule: It refers to the rule of maintaining the order in which jobs are to be processed on given machines.

Assumptions:

1. The processing time on different machine are machines are exactly known and are independent of the order of the jobs in which they are to be processed.
2. The time taken by the job in moving from one machine is negligible.
3. Once a job has begun on a machine, it must be completed before another job can begin on the same machine.

4. All jobs are known and are ready for processing before the period under consideration begins.
5. Only one job can be processed on a given machine at a time.
6. Machines to be used are of different types.
7. The orders of completion of jobs are independent of he sequence of jobs.

2.2 Processing n Jobs through two machines

Let there be n jobs, each of which is to be processed through two machines. M_1 and M_2 in the order M_1 M_2 i.e. each job has to be pass through the same sequence of operations. In other words a job is assigned to machine M_1 first and after it has been completely processed on machine M_1, it is assigned to machine M_2. If the machine M_2 is not free at the moment for processing the same job, them the job has to wait in a waiting line for its turn on machine M_2, so passing is not allowing.

Since passing is not allowed, therefore machine M_1 will remain busy in processing all the n jobs one-by-one while machine M_2 may remain idle time of the second machine. This can be achieved only by determining sequence of n jobs which are to be processed on two machines M_1 and M_2. The procedure suggested by Johnson for determining the optimal sequence can be summarized as follows.

The Algorithm

Step 1 List the jobs along with their processing times in a table as shown below:

Processing Time on Machine	Job Number				
	1	2	3	....	N
M_1	t_{11}	t_{12}	t_{13}	....	t_{1n}
M_2	T_{21}	T_{22}	T_{23}	....	T_{2n}

Step 2 Examine the columns for processing times on machines M_1 and M_2, and find the smallest processing time in each column, i.e. find out, min(t_{1j}, t_{2j}) for all j.

Step 3(a) If the smallest processing time is for the first machine M_1, then place the corresponding job in the first available position in the sequence. If it is for the second machine, the place the corresponding job in the last available position in the sequence.

(b) If there is a tie in selecting the minimum of all the processing times, then there may be three situations:

(i) Minimum among all processing times is same for the machine, i.e. min (t_{1j}, t_{2j})= t_{1k} = t_{2r}, then process the kth job first and the rth job last.

(ii) If the tie for the minimum occurs among processing times t_{1j} on machine M_1 only, then select the job corresponding to the smallest job subscript first.

(iii) If the tie for the minimum occurs among processing time t_{2j} on machine M_2, then select the job corresponding to the largest job corresponding to the largest job subscript last.

Step 4 Remove the assigned jobs from the table. If the table is empty, stop and go to step 5. Otherwise, go to Step 2.

Step 5 Calculate idle time for machines M_1 and M_2:

(a) Idle time for machine M_1 = (Total elapsed time)-(Time when the last job in a sequence finishes on machine M_1)

(b) Idle time for machine M_2 = Time at which the first job in a sequence finishes on machine M_1

$$+\sum_{j=2}^{n} \{\text{Time when the job in a sequence starts on Machine } M_2\}$$

-{Time when the (j-1)th job in a sequence finishes on Machine M_2}

Step 6 The total elapsed time to process all jobs through two machines is given by

Total elapsed time = Time when the nth job in a sequence finishes on machine M_2.

$$= \sum_{j=1}^{n} M_{2j} + \sum_{j=1}^{n} I_{2j}$$

Where M_{2j} = Time required for processing jth job on machine M_2.

I_{2j} = Time for which machine M_2 remains idle after processing (j-1)th job and before starting work in jth job.

2.3 Processing n Jobs through three machines

Johnson provides an extension of his procedure to the case in which there are three instead of two machines, each job is to be processed through three machines M_1, M_2 and M_3 in the order M_1, M_2 M_3. The list of jobs with their processing times is given below. An optimal solution to this problem can be obtained if either or both of the following conditions hold good:

Processing Time on Machine	Job Number 1	2	3	….	N
M_1	t_{11}	t_{12}	t_{13}	….	t_{1n}
M_2	T_{21}	T_{22}	T_{23}	….	T_{2n}
M_3	T_{31}	T_{32}	T_{33}	….	T_{3n}

1. The minimum processing time on machine M_1 is at least as great as the maximum processing time on machine M_2, that is,

$$\min t_{1j} \geq \max t_{2j}, \qquad j = 1, 2, \ldots\ldots, n$$

2. The minimum processing time on machine M_3 is at least as great as the maximum processing time on machine M_2, that is,

$$\min t_{3j} \geq \max t_{2j}, \qquad j = 1, 2, \ldots\ldots, n$$

If either or both the above conditions hold good, then the steps of the algorithm can be summarized in the following steps.

The Algorithm

Step 1 Examine processing time of given jobs on all three machines and if either one or both the above conditions hold, then go to Step 2, otherwise the algorithm fails.

Step 2 Introduce two fictitious machines, say G and H with corresponding processing times given by

G_i and H_i are defined by $G_i - A_i + B_i$, $H_i = B_i + C_i$

Then applying the Johnson's job 2 machine algorithm

2.4 Processing n jobs through m machines

Let there be n jobs, each of which is to be processed through m machines, say M_1, M_2, ... M_m in the order M_1 M_2 ... M_m. The optimal solution to this problem can be obtained if either or both of the following condition hold good.

(a) Min $\{t_{1j}\} \geq$ Max $\{t_{ij}\}$; j = 2, 3, ..., m-1

and/or (b) Min $\{t_{mj}\} \geq$ Max $\{t_{ij}\}$; j = 2, 3, ..., m-1

that is, the minimum processing time on machines M_1 and M_m is as great as the maximum processing time on any of the remaining (m-1) machines.

If either or both these conditions hold good, then the steps of the algorithm can be summarized in the following steps.

Step 1 Find, Min $\{t_{1j}\}$, Min$\{t_{mj}\}$ and Max$\{t_{ij}\}$ and verify above conditions. If either or both the conditions mentioned above hold, then go to Step 2. Otherwise the algorithm fails.

Step 2 Convert m-machine problem into 2-machine problem by introducing two fictitious machines, say G and H with corresponding processing times given by

(i) $t_{Gj} = t_{ij} + t_{2j} + \ldots + t_{m-1j}$; j= 1, 2, ..., n

i.e. processing time of n-jobs on machine G is the sum of the processing times on machines M_1, M_2, ...$M_{m-1,j}$

(ii) $t_{Hj} = t_{2j} + t_{3j} + \ldots + t_{mj}$; j= 1, 2, ..., n

i.e. processing time of n-jobs on machine H is the sum of the processing times on machines M_1, M_2, ...$M_{m-1,j}$

Step 3 The new processing times so obtained can now be used for solving n-job, two-machine equivalent sequencing problem with the prescribed ordering HG in the same way as discussed earlier.

2.5 Concept of equivalent job in flow shop

The most common optimizer in flow shop requiring has been to minimize the total elapsed time (make span or maximum flow time) for a set of n independent jobs to be processed over m ordered machines. Now consider a situation in which some sets of specified jobs are required to be processed together as a block in a sequence either by virtue of technological constraint or some externally imposed restriction. This type of situation is known as a **Group Technology** which has very wide applications to a variety of production systems for the purpose of improving the productivity. The problem of determining an optimal sequence under the stated restriction is difficult to be stored with the help of available method.

2.6 Equivalent job block theorem given by Maggu & Das

Let there be two jobs i and j is a sequence S to be processed on two machines A and B in the order A B. Let the equivalent job of i and j b denoted by a'

Then $A_a = A_i + A_j - \min(A_j, B_i)$

$B_a = B_i + B_j - \min(A_j, B_i)$

Where A_a and B_a denote the processing time of equivalent job 'a' on machine A and B respectively.

2.7 Concept of Transportation time in flow shop:

In many practical situations of scheduling it is seen that machines are distantly situated and therefore, definite finite time is taken in transporting the job from one machine to another if the form of.

i) Loading time of jobs.

ii) Moving time of jobs.

iii) Unloading time of jobs.

The sum of all the above times has been designated by various researches as transportation time of a job. This transportation time is the amount of time required to dispatch the job i after it has been completed on machine A, to the next succeeding machine B for its onward processing. It is denoted by t_i for job i.

2.8 Flow Shop problem with break down of machines

it has been assumed so for that no machine fails and hence no disturbance occurs in the processing of the jobs. Many a times it is practically possible that machine may not work:

i) Due to failure of electric supply from mains. Or

ii) Machines stop working due to failure of one or more components suddenly.

iii) Machines are required to stop for certain interval of time due to excessive leading or some other external course. Therefore now we are considered to take into account the effect of break down interval of machines on the completion time of the set of jobs. Now we presses a heuristic method for providing an optimal or near optimal solution for a n x 2 flow shop problem.

Step of the Algorithm

Step 1: Determine the optimal sequence so for the given n x 2 flow shop problem by Johnson's algorithm.

Step 2: Find the optimal total elapsed time for the sequence so

Step 3: Locate the time intervals for jobs r.t the break down intervals overlaps with these. Two cases are possible.

i) Either the break down interval is not present practically or fully in any time interval of the completion of jobs on any machine. In this cases effect of the break-down on the total elapsed time, and sequence so obtained in step 1 is optimal stop the process.

ii) If the break down interval is contained practically or fully in some of the completion time intervals of jobs. Go to step 4.

Step 4: Formulate a new problem with processing time A'_i and B'_i for job i on machine A and B respectively s.t

$$A'_i = A_i + (b-a)$$

$$B'_i = B_i + (b-a)$$

Where (b-a) represents the break down time interval of machines.

Step 5: Determine the optimal sequence by Johnson's algorithm for the modified problem obtained in step 4. This new sequence is either new optimal or near optimal for the original problem. Also determine the total elapsed time.

2.9 n-jobs two machines flow shop with equivalent job for a job block transportation time for a job and break down

This topic gives a heuristic approach to study n-job, 2-machine flow-shop problem in which equivalent job for job-block; transportation time for a job from one machine to another machine and break down machine time was involved. The objective of the chapter is to find optimal or near optimal solution technique according to which processing of sequence gives the minimum or near to minimum total completion times for n jobs in the sequence.

Johnson (1954) basically studied a two machine n-job flow shop problem and he gave an optimal algorithm to find a sequence giving minimum completion time for all jobs when processed through two machines. He, further extended the optimal algorithm for a special type of three machine flow shop problem. After this study, a good deal of efforts has been made of three machine flow shop problem. After this study, a good deal of efforts has been made to find an optimal solution for the general n job m machine flow shop problem. However, various research cal attempts have been made to find near optimal solution for a general type of flow shop problem through heuristic approaches or numerical techniques. To mention a few of the attempts made by the researcher will refer to studies by Mitten, Maggu, G. Das etc. Further mention in the above studies it was assumed that the transportation time of a job after completion on one machine and then going to the successive machine is negligible. Here we have taken into account the concept of transportation time.

PROBLEM:

n jobs are processed through two machines A and B in the order (A, B) with processing times for job i on machines A and B being defined as A_i and B_i respectively. Let t_i be transportation time of job i from machine A to B. Let (a, b) be the break down interval with length I = b-a. Let the jobs be performed in a job block $\beta = (\alpha_1, \alpha_2)$ where α_1, α_2 are, out of n jobs 1, 2, 3, ……., n.

Then problem is to find an optimal or near optimal sequence to minimize the total elapsed time in completion of all jobs on the two machines.

Step [1]: Find a reduced problem with new processing time A_i' and B_i' defined as

$A_i' = A_i + t_i$

$B_i' = B_i + t_i$

On the lines of Step [1] in Maggu and Das [1980] problem.

Step [2]: Find processing times for the equivalent job β following Maggu and Das [1977] for the problem in Step [2] as follows:

$A'_\beta = A'_\alpha + A'\alpha_{k+1} - \min(\beta'\alpha_k, A'\alpha_{k+1})$

$B'_\beta = B'_\alpha + B'\alpha_{k+1} - \min(B'\alpha_k, A'\alpha_{k+1})$

Step [3]: Now form a reduced problem replacing the given job-block in the theorem (as per Step [1]) by their equivalent job.

Step [4]: Use Johnson's method to obtain the optimal sequence of reduced problem as per steps and read effect of break down interval of (a, b) on different jobs.

Step [5]: Find a reduced problem in Step [3] with processing times A_i'' B_i''

Where

$$\left.\begin{aligned} A_i'' &= A_i' + 1 \\ B_i'' &= B_i + 1 \end{aligned}\right\} \text{ if (a, b) has effected on job i}$$

Where

$$\left.\begin{aligned} A_i'' &= A_i' \\ B_i'' &= B_i \end{aligned}\right\} \text{ if (a, b) has no effect on job i}$$

Step [6]: Now repeat the procedure to get optimal sequence of reduced problem as Step [5] using step [2], [3], [4] on the similar lines as done for getting the optimal sequence of reduced problem as per Step [1].

Then one of the optimal sequences is optimal or near optimal for the original problem.

Numerical Example:

Let the problem be defined as, job processing times of job

(i)	A (Ai)	B (Bi)	Transportation Times (ti)
1	6	2	3
2	7	8	4
3	11	6	6
4	5	11	2

with equivalent job β= (1, 4) and with break down interval (a, b) defined by (a, b) = (13, 20) with l= 20-13= 7

Problem is to find optimal or near optimal sequence summing the total elapsed time.

Solution: Reduced problem as per step [1] is,

Job (i)	A': $(A_i + t_i)$	B': $(B_i + t_i)$
1	9	5
2	11	12
3	17	12
4	7	13

Processing time for equivalent job β= (1, 4) as per step [2] is

$A'\beta = A'1 + A'4 - \min(B'1, A'4)$
$= 9 + 7 - \min(5, 7)$
$= 16 - 5$
$= 11$

$B'\beta = B'1 + B'4 - \min(B'1, A'4)$
$= 5 + 13 - \min(5, 7)$
$= 18 - 5$
$= 13$

Reduced problem as per step [3] is as follows:

Job	A';	B';
B	11	13
2	11	12
3	17	12

Using Johnson's method optimal sequence of problem as per step is either (β, 2, 3) or (2, β, 3) replacing equivalent job β for job-block (1, 4) optimal sequences are (, 4, 2, 3) or (2, 1, 4, 3) now two cases arises as follows:

Case[1]: Effect of break-down interval (13, 20) on sequence (1, 4, 2, 3) is read as follows:

Job	A	ti	B
	in-out		in-out
1	0 – 6	3	9 – 11
4	6 – 11	2	13 – 24
2	11 – 18	4	24 – 35
3	18 – 29	6	35 - 41

Reduced problem as per step [5]

Job	A';	B';
1	6	2
2	7	8
3	11	6
4	5	18

$$\begin{aligned} A''\beta &= A''1 + A''4 - \min(B''1, A''4) \\ &= 6 + 5 - \min(2, 5) \\ &= 11 - 2 \\ &= 9 \end{aligned}$$

$$\begin{aligned} B''\beta &= B''1 + B''4 - \min(B''1, A''4) \\ &= 2 + 18 - \min(2, 5) \\ &= 20 - 2 \\ &= 18 \end{aligned}$$

Reduced problem is

Job (i)	A'i	B'i
β	9	18
2	7	8
3	11	6

Using Johnson' technique the optimal sequence is (β, 2, 3) or (2, 1, 4, 3)

Job(i)	A	ti	B
2	0 – 7	4	11 – 19
1	7 – 13	3	19 – 21
4	13 – 18	2	21 – 32
3	18 – 29	6	35 - 41

Reduced problem in this case is as per step [5]

Job (i)	A'i	B'i
1	6	9
2	7	8
3	18	6
4	12	11

$$A''\beta = A''1 + A''4 - \min(B''1, A''4)$$
$$= 6 + 12 - \min(9, 12)$$
$$= 18 - 9$$
$$= 9$$

$$B''\beta = B''1 + B''4 - \min(B''1, A''4)$$
$$= 9 + 11 - \min(9, 12)$$
$$= 20 - 9$$
$$= 11$$

The further reduced problem is

Job	A'i	B'i
β	9	11
2	7	8
3	18	6

Using Johnson's technique, the optimal sequence is (2, β, 3) or (2, 1, 4, 3)

The optima or near optimal sequence is either (1, 4, 2, 3) or (2, 1, 4, 3). Since in each case the total minimum elapsed time is 47 hours therefore, each of the two sequences is optimal.

2.10 On Equivalent job for job block in a three machine flow shop problem with break down machine times included

This chapter has the objective to give heuristic algorithm for finding an optimal or near optimal solution minimizing the total elapsed time in nx3 Johnson's flow shop for block is taken into account when the machines operating the jobs are allowed to got break downs for certain times of intervals. The efficiency of the heuristic algorithm (found on the basis of experimental work) is verified by means of an illustrative example.

Heuristic Algorithm:

The heuristic algorithm scanning optimal or near optimal schedule minimizing the total elapsed time may consist of the following steps:

Step [1]: Find the reduced problem by replacing the block job processing times by their respective equivalent jobs processing times with the aid of Das [1978] or Maggu, Das and Sishpal [1982] methods.

Step [2]: Find the effect of break down interval on the flow time intervals of the different jobs for the reduced problem in step [1].

Step [3]: Find a second reduced problem in which processing times are defined by A_i', B_i', C_i' as

$$\left.\begin{aligned} A_i'' &= A_i' + 1 \\ B_i'' &= B_i + 1 \\ C_i'' &= C_i + 1 \end{aligned}\right\} \quad \text{if job i is effected by break down interval (a, b)}$$

$$\left.\begin{aligned} A_i'' &= A_i' \\ B_i'' &= B_i \\ C_i'' &= C_i \end{aligned}\right\} \quad \text{if job i is effected by break down interval (a, b)}$$

Step [4]: Find the equivalent job processing time for the given job block in reduced as per Step [3].

Step [5]: Find a third reduced problem replacing the given set of jobs by their equivalent jobs with processing times as calculated in the above Step [4].

Step [6]: Use Johnson's technique to find optimal sequence the problem in Step [5].

Step [7]: One of the optimal sequences obtained in step [6] is now optimal or ner optimal sequence for the original problem.

Illustrative Example:

To illustrate the procedure of the heuristic principle, we now take up one numerical problem in the tableau form as follows:

Jobs		**Machines**	
(i)	A	B	C
	(t_i1)	(t_i2)	(t_i3)
1	15	8	8
2	10	9	7
3	11	10	7
4	14	9	9
5	12	10	6
6	13	7	5
7	11	6	4

The problem is to obtain optimal sequence in which job β is equivalent job for an order pair job block (1, 5) of job 1 and job 5 in this order with breakdown intervals (35, 47) with l=12.

Solution: here,

Min (t_{is}) ≥ max (t_{is+1}) (S= 1, 2, 3)

Are satisfied also here,

$\alpha_k = \alpha_{k=1} = 5$

As per step [1], the processing times of β on machine A, B, C are given by using the formal due to Maggu, Das and Shishpal [1982]

$t_{\beta A}$ = $(t_{1A} = U_1) + (t_{5A} + U_5) - \min (t_{ic} + u_i)\ t_{5A} = U_5)$

= 15 + 8 + 12 + 10 – min (8 + 8, 12 + 10)

= 23 + 22 – 16

= 29

$t_{\beta\beta} = 0$

$t_{\beta C}$ = $(t_{1C} = U_1) + (t_{5C} + U_5) - \min (t_{ic} + u_i)\ t_{5A} = U_5)$

= 8 + 8 + 6 + 10 – min (8 + 8, 12 + 10)

= 16

Hence, by step [2], replacing jobs (1, 5) by β reduces the problem into

Jobs		**Machines**	
(i)	A	B	C
β	29	0	16
2	10	9	7
3	11	10	7
4	14	9	9
6	13	7	5
7	11	6	4

Using Johnson's technique, the optimal sequence is

4	3	2	β	6	7

The flow- time chart for the sequence 4, 3, 2, β, 6, 7 with replacing β by (1, 5) is

Job	**A**	**B**	**C**
(i)	in-out	in-out	in-out
4	0 – 17	14 – 23	23 – 32
3	14 – 25	25 – 35	35 – 42
2	25 – 35	35 – 44	44 – 51
1	35 – 50	50 – 58	58 – 60
5	50 – 62	62 – 72	72 – 78
6	62 – 75	75 – 82	82 – 87
7	75 – 86	86 – 92	92 - 96

Now using Maggu's technique [1982] after reading the effects of the break down interval (35, 47) we have the reduced problem as per step [3].

Job	**A**	**B**	**C**
(i)	in-out	in-out	in-out
4	14	9	9
3	11	10	19
2	10	21	7
1	27	8	8
5	12	10	6
6	13	7	5
7	11	6	4

$t_{\beta A}$ $= (t_{1A} = u_1) + (t_{5A} + u_5) - \min\,(t_{1c} + u_1)\ t_{5A} = u_5)$

$= 27 + 8 + 12 + 10 - \min\,(8 + 8,\ 12 + 10)$

$= 41$

$t_{\beta\beta} = 0$

$t_{\beta C}$ = 8 + 8 + 12 + 10 – 16
= 22

As per step [5] the reduced problem becomes

Job	**A**	**B**	**C**
(i)	(A_i)	(B_i)	(C_i)
4	14	9	9
3	11	10	19
2	10	21	7
β	41	0	22
7	11	6	4

Using Johnson's technique, optimal sequence is

3	4	2	β	6	7

Or 3, 4, 2, 1, 5, 6, 7 when β is replaced by (1, 5). This optimal sequence (3, 4, 2, 1, 5, 6, 7) is also optimal or near optimal for the original problem. The flow time chart in the sequence 3, 4, 2, 1, 5, 6, 7 job is given below

Job	**A**	**B**	**C**
(i)	in-out	in-out	in-out
3	0 – 11	11 – 21	21 – 28
4	11 – 25	25 – 34	34 – 43
2	25 – 35	35 – 44	44 – 51
1	35 – 50	50 – 58	58 – 66
5	50 – 62	62 – 72	72 – 78
6	62 – 75	75 – 82	82 – 87
7	75 – 86	86 – 92	92 - 96

With minimum total elapsed time is 96 hours.

2.11 Minimizing Rental Cost under specified rental policy in Two Stage Flow Shop

Recently an attempt has been made by T.P. Singh et al. [2006] to study the two machine general flow shop problem following some restrictive renting policy including equivalent job-block criteria. The object of the study is to find an algorithm to minimize the rental cost of the machines under specified renting policy.

2.12 PRACTICAL SITUATIONS

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the ultra sound machine etc. but instead takes on rent. The examination branch of a board/institute needs machines as data entry machine, computer, printer etc. on rent for computerizing and compiling examination result for secrecy point of view.

Moreover in hospitals industries concern, sometimes the priority of one job over other is preferred. It may be because of urgency or demand of its relative importance. Hence the job block criteria becomes significant.

2.13 NOTATIONS

S: Sequence of job 1, 2, 3, ……., n

M_j : Machine j, j= 1, 2, ……….

A_i: Processing time of i^{th} job on machine A.

B_i: Processing time of i^{th} job on machine B.

A'_i: Expected processing time of i^{th} job on machine A.

B'_i: Expected processing time of i^{th} job on machine B.

p_i: Probability associated to the processing time A_i of i^{th} job on machine A.

q_i: Probability associated to the processing time B_i of i^{th} job on machine B.

β: Equivalent job for job-block.

S_i: Sequence obtained from Johnson's procedure to minimize rental cost.

C_j: Rental cost per unit time of machine j.

U: Utilization time B (2^{nd} machine) for each sequence S_i.

$t_1(S_i)$: Completion time of last job of sequence S_i on machine A.

$t_1(S_i)$: Completion time of last job of sequence S_i on machine A.

$R(S_i)$: Total rental cost for sequence S_i of all machines.

$CT(S_i)$: Completion time of 1^{st} job of each sequence S_i on machine A.

2.14 ASSUMPTIONS

1. We assume rental policy that all the machines are taken on rent as and when they are required and are returned as when they are no longer required for processing. Under this policy second machine is taken on rent at time when first job completes its processing on first machine. Therefore idle time of second machine for first job is zero.

2. Jobs are independent to each other.
3. Machine break down is not considered. This simplifies the problem by ignoring the stochastic component of the problem.
4. Per-emption is not allowed i.e. jobs are not being split, clearly, once a job is started on a machine, the process on that machine can't be stopped unless the job is completed.

2.15 DEFINITIONS

Definition 1

As operation is defined as a specific job on a particular machine.

Definition 2

Sum of idle time of M_2 (for all jobs)

$$\sum_{i=1}^{n} Ii2 = \max [\sum_{i=1}^{n} A'_i - \sum_{i=1}^{n-1} B'_i, \sum_{i=1}^{n-1} A'_i - \sum_{i=1}^{n-2} B'_i \sum_{i=1}^{n-2} A'_i - \sum_{i=1}^{n-3} B'_i ....\sum_{i=1}^{2} A'_i - \sum_{i=1}^{2-1} B'_i A'_i]$$

$= \max [P_n, P_{n-1} ....P_2, P_1]$

$= \max [P_k]$

where

$$P_k = \sum_{i=1}^{k} A'_i - \sum_{i=1}^{k-1} B'_i$$

$1 \le k \le n$ $\quad$ i=1

and $A'_i = A_i \times p_i$

$B'_i = B_i \times q_i$

Definitions

Total elapsed time for a given sequence.

= Sum of expected processing time on 2^{nd} machine (M_2)+ total idle time on M_2.

$$= \sum_{i=1}^{n} B'_i + \sum_{i=1}^{n} I'_{i2}$$

$$= \sum_{i=1}^{n} B'_i + \max [P_k], \text{ where } P_k = \sum_{i=1}^{n} A'_i - \sum_{i=1}^{n} B'_i$$

Note 1:

Idle time of 1^{st} machine is always zero i.e. $\sum_{i=1}^{n} I'_{i1} = 0$

Note 2:

Idle time of 1^{st} job on 2^{nd} machine i.e. I^{i2} = Expected processing time of 1^{st} job on 1^{st} machine.

$= A'_1$

Note 3:

Rental cost of machines will be minimum if idle time of machine 2 is minimum.

2.16 Now we state two theorems which are applied in our algorithm

Theorem 1:

Equivalent job block theorem due to Maggu & Das [1977]. In two machine flow shop problem “In processing a schedule S = (α_1, α_2, ………….. α_{k-1}, ………. α_n) of n jobs on two machines A and B in the order AB with no passing allowed the job block (α_k, α_m) having processing time $A\alpha_k$, $B\alpha_k$, $A\alpha_m$, $B\alpha_m$ is equivalent to the single job β (called equivalent job β). The processing times of equivalent job β on the machines. A & B denote respectively by A_β and B_β are given by.

$A_\beta = A_{\alpha k} + A_{\alpha m} - \min(B_{\alpha k}, A_{\alpha m})$

$B_\beta = B_{\alpha k} + B_{\alpha m} - \min(B_{\alpha k}, A_{\alpha m})$

Theorem 2:

Job i precedes to job j in optimal ordering having idle time on B. If $\min(A'_i, B'_j) < \min(A'_j, B'_i)$.

Where

A'_i = Expected processing time of i^{th} job on A = $A_i \times p_i$

B'_i = Expected processing time of i^{th} job on B = $B_i \times q_i$

Algorithm

Step 1: Define expected processing time of job block β = (k, m) on machine A & B using equivalent job block Criteria given by Maggu & Das [6] i.e. find A'_β & B'_β as:

$A'_\beta = A'_k + A'_m - \min(B'_k, A'_m)$

$B'_\beta = B'_k + B'_m - \min(B'_k, A'_m)$

Step 2: Using Johnson's two machine algorithm [5] obtain the sequence S_i, which minimize the total elapsed time.

Step 3: Observe the processing time of 1^{st} job of S_1 on the first machine A: let it be α

Step 4: Obtain all the jobs having processing time on A greater than α. Put these jobs one by one in the 1^{st} position of the sequence S_1, keeping other jobs of the sequence S_1 in the same order. Let these sequence S_2, S_3, S_4, …………., S_n.

Step 5: Prepare in-out table for each sequence S_1 (i= 1, 2, ……….r) and evaluate total completion time of last job of each sequence machine $t_1(S_i)$ & $t_2(S_i)$ on machine A & B respectively.

Step 6: Evaluate completion time CT (S_i) of 1st job of each sequence S_i on machine A.

Step 7: Calculate utilization time U_i of 2nd machine for each sequence S_i as

$U_i = t_2 (S_i) – CT (S_i)$ for i= 1, 2, 3,……..

Step 8: Find Min {U_i}, i= 1, ……r. Let it be corresponding to i=m. then S is the optimal sequence for minimum rental cost.

Min Rental Cost = $t_1 (S_m) \times C_i + U_m \times C_2$

Where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

Numerical Illustration

Consider 5 jobs and 2 machines problem to minimize the rental cost. The processing times are given as follows:

Jobs	**Machine A** (A_i)	**Machine B** (B_i)
1	11	7
2	15	11
3	12	13
4	17	16
5	14	17

Rental costs per unit time for machine A & B are 15 and 13 units respectively, and jobs 2, 5 are to be processed as an equivalent group job β.

Solution: Jobs by alongwith the processing times of equivalent job block β = (2, 5) are given by

A'_β = 15 + 14 – 11

= 18

B'_β = 11 + 17 – 11

= 17

Jobs	**A_i'**	**B_i'**
1	11	7
β	18	17
3	12	13
4	17	10

As per step II using Johnson's method, optimal sequence is

S_1 = 3, β, 4, 1

i.e. 3, 2, 5, 4, 1

S_2 = 1, 4, 3, 2, 5

S_3 = 2, 5, 4, 3, 1

S_4 = 2, 5, 4, 1, 3

These sequences are enumerated in following table

S_1 = 3, 2, 5, 4, 1

Jobs	A	B
	In-Out	In-Out
3	0 – 12	12 – 25
2	12 – 27	27 – 38
5	27 – 41	41 – 58
4	41 – 58	58 – 74
1	58 – 69	74 - 81

Then the total elapsed time = 81 units and utilization time for B = 81 – 12 = 69 units

S_2 = 1, 4, 3, 2, 5

Jobs	A	B
	In-Out	In-Out
3	0 – 11	11 – 18
4	11 – 28	28 – 44
3	28 – 40	44 – 57
2	40 – 55	57 – 68
5	55 – 69	69 - 86

Total elapsed time = 86 units utilization of B = 86 – 11 = 75 units

S_3 = 2, 5, 4, 3, 13

Jobs	A	B
	In-Out	In-Out
2	0 – 15	15 – 26
5	15 – 29	29 – 46
4	29 – 46	46 – 62
3	46 – 58	62 – 75
1	58 – 69	75 - 82

Total elapsed time=82 units utilization time of B=82–15=67 units

S_4 = 2, 5, 4, 3, 13

Jobs	A	B
	In-Out	In-Out
2	0 – 15	15 – 26
5	15 – 29	29 – 46
4	29 – 46	46 – 62
3	46 – 57	62 – 69
1	57 – 69	69 - 82

Total elapsed time=82 units utilization time of B=82–15=67 units

The total utilization of A machine is fixed 69 units and minimum utilization time of B machine is 67 units for two sequence S_3 and S_4. Therefore optimal sequence are S_3 2-5-4-3-1 and S_4 2-5-4-1-3 and total rental cost = 15 x 69 + 13 x 67

= 1035 + 871

= 1906 units

Conclusion

The study may further by extended if parameter like set up time, transportation time etc. are taken into consideration.