Unit: 3

Numerical Integration

$$\int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_n} y\,dy \quad = f(x) = y$$

Function of $f(x)$ integrate w.r.t x of the limit x_0 to x_n. x_0 is the lower limit. x_n is the final limit ie upper limit.

$$\boxed{x_n = x_0 + nh}$$

n is the total no of steps and h is the equal interval.

There are 3 methods to find the numerical integration of any function

1) Trapezoidal Rule // any no of n
2) Simpson's 1/3 Rule // Even no. of n
3) Simpson's 3/8 Rule // multiple of 3 of n

$x_n = x_0 + nh$

if $n = 4$

$1 = 0 + 4h$

$h = 0.25$

$x_1 = x_0 + h$

$x_2 = x_0 + 2h$

$x_3 = x_0 + 3h$

$x_0 = y_0 = x_0^2 = 0$
$x_1 = y_1 = x_1^2 = (.25)^2$
$x_2 = y_2 = x_2^2$

General formula of Trapezoidal Rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left\{[y_0 + y_n] + 2(y_1 + y_2 + y_3 \dots y_{n-1})\right\}$$

if $(n = 4)$, then $\frac{h}{2}\left[(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right]$

Q $\int_0^1 \frac{dx}{1+x}$, $n = 4$ by Trapezoidal Rule.

Sol

$x_0 = 0 \qquad x_n = 1$

$x_n = x_0 + nh$

$1 = 0 + 4h$

$h = 0.25$

$x_1 = x_0 + h$

$= 0 + .25 = \boxed{.25}$

$x_2 = x_0 + 2h$

$\Rightarrow 0 + 2(.25)$

$\Rightarrow \boxed{.5}$

$x_0 = 0 \qquad y_0 = \frac{1}{1+x_0} = \frac{1}{1+0} = 1$

$x_1 = .25 \qquad y_1 = \frac{1}{1+x_1} = \frac{1}{1+.25} = 0.8$

$x_2 = .5 \qquad y_2 = \frac{1}{1+x_2} = \frac{1}{1+.5} = .67$

$x_3 = .75 \qquad y_3 = \frac{1}{1+x_3} = \frac{1}{1+.75} = 0.571$

$x_4 = 1.0 \qquad y_4 = \frac{1}{1+x_4} = \frac{1}{1+1} = 0.5$

$$I = \int_0^1 \frac{dx}{1+x} = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3)\right]$$

$$\Rightarrow \frac{0.25}{2}\left[(1+0.5) + 2(0.8 + .67 + 0.571)\right]$$

$$\Rightarrow (0.125)\left[(1.5) + (4.082)\right]$$

$$\Rightarrow \boxed{0.697}$$

$$\int_0^1 \frac{dx}{1+x} = \left[\log(1+x)\right]_0^1$$

$$= \log 2 - \log 1 = \log 2 - \log 0$$

$$= \boxed{0.693}$$

Difference is $0.697 - 0.693$

$$= \boxed{0.004}$$

Q $\int_1^3 \frac{1}{x}\,dx$ using n.s and compare to exact integral

$= \int_1^3 \frac{1}{x}\,dx$

$x_n = x_0 + nh$

$3 = 1 + 5h$

$\boxed{h = 0.4}$

$x_1 = x_0 + h$

$= 1 + 0.4 = \boxed{1.4}$

$x_2 = x_0 + 2h$

$= 1 + 2(0.4) = \boxed{1.8}$

$x_3 = x_0 + 3h$

$= 1 + 3(0.4) = \boxed{2.2}$

$x_4 = x_0 + 4h$

$= 1 + 4(0.4) = \boxed{2.6}$

$x_5 = x_0 + 5h$

$= 1 + 5(0.4) = \boxed{3}$

$y_0 = \frac{1}{x_0} = \frac{1}{1} = 1$

$y_1 = \frac{1}{x_1} = \frac{1}{1.4} = \boxed{0.71}$

$y_2 = \frac{1}{x_2} = \boxed{0.55}$

$y_3 = \frac{1}{x_3} = \frac{1}{2.2} = \boxed{0.454}$

$y_4 = \frac{1}{x_4} = \frac{1}{2.6} = \boxed{0.384}$

$y_5 = \frac{1}{x_5} = \frac{1}{3} = \boxed{0.333}$

$\Rightarrow \frac{0.4}{2}\left[(1+0.33) + 2(0.71 + 0.55 + 0.454 + 0.384)\right]$

$= 0.2\,(1.333 + 4.196)$

$= \boxed{1.1058}$

2) Simpson's 1/3 Rule

(n → even)

$$\int_{x_0}^{x_n} y\,dx = \frac{1}{3}h\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots+y_{n-1})+2(y_2+y_4+\cdots y_{n-2})\right]$$

if n = 6

$$= \frac{1}{3}h\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

3) Simpson's 3/8 Rule

(n → multiple of 3)

if $n = 6$

$$\frac{3}{10} h \left[(y_0 + y_6) + 5(y_1 + y_2 + y_4 + y_5) + 2(y_3) \right]$$

Q $\int_4^{5.2} \log x \, dx$ by using of three method $n = 6$

$$\int_4^{5.2} \log x \, dx$$

$x_n = x_0 + nh$

$5.2 = 4 + 6h$

$\Rightarrow \boxed{0.2}$

$x_0 = 4$

$x_1 = x_0 + h$
$= 4 + 0.2$
$\Rightarrow 4.2$

$x_2 = x_0 + 2h$
$\Rightarrow 4 + 2(0.2)$
$\Rightarrow 4.4$

$x_3 = x_0 + 3h$
$= 4 + 3(0.2)$
$\Rightarrow 4.6$

$x_4 = x_0 + 4h$
$= 4 + 4(0.2)$
$\Rightarrow 4.8$

$x_5 = x_0 + 5h$
$\Rightarrow 4 + 5(0.2)$
$\Rightarrow 5$

$x_6 = x_0 + 6h$
$\Rightarrow 4 + 6(0.2)$
$\Rightarrow 5.2$

if $n=6$

$$=\frac{3}{8}h\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

Q $\int_4^{5.2}\log x\,dx$ by using of three method $n=6$

$$\int_4^{5.2}\log x\,dx$$

$x_n = x_0 + nh$

$5.2 = 4 + 6h$

$\Rightarrow \boxed{0.2}$

$x_0 = 4$

$x_1 = x_0 + h$
$= 4 + 0.2$
$\Rightarrow 4.2$

$x_2 = x_0 + 2h$
$\Rightarrow 4 + 2(0.2)$
$\Rightarrow 4.4$

$x_3 = x_0 + 3h$
$= 4 + 3(0.2)$
$\Rightarrow 4.6$

$x_4 = x_0 + 4h$
$= 4 + 4(0.2)$
$\Rightarrow 4.8$

$x_5 = x_0 + 5h$
$\Rightarrow 4 + 5(0.2)$
$\Rightarrow 5$

$x_6 = x_0 + 6h$
$\Rightarrow 4 + 6(0.2)$
$\Rightarrow 5.2$

$+y_5$

$y_0 = \log x_0 = \log 4 = 1.386$

$y_1 = \log x_1 = \log 4.2 = 1.435$

$y_2 = \log x_2 = \log 4.4 = 1.482$

$y_3 = \log x_3 = \log 4.6 = 1.526$ (2)

$y_4 = \log x_4 = \log 4.8 = 1.568$

$y_5 = \log x_5 = \log 5 = 1.609$

$y_6 = \log x_6 = \log 5.2 = 1.648$

(i) Trapezoidal Rule

$$\Rightarrow \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$\Rightarrow \frac{0.2}{2}\left[(1.386 + 1.648) + 2(1.435 + 1.482 + 1.526 + 1.568 + 1.609)\right]$$

$\cancel{0.1(3.034 + 18.536)}$

$\Rightarrow \boxed{1.827}$

(2) Simpson's 1/3 Rule

$\frac{1}{3}h\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$\Rightarrow \frac{0.2}{3}\left[(1.386 + 1.648) + 4(1.435 + 1.526 + 1.609) + 2(1.482 + 1.568)\right]$

$\frac{0.2}{3}\left[3.034 + 4(4.57) + 2(3.05)\right]$

$\Rightarrow \frac{0.2}{3}\left[3.034 + 18.28 + 6.1\right]$

$\Rightarrow \boxed{1.827}$

(3) Simpson's 3/8 Rule

$$\frac{3}{8}h\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$\frac{3}{8}(0.2)\left[3.034+3(1.435+1.482+1.609+1.568)+2(1.526)\right]$$

$$\frac{3}{8}(0.2)\left(3.034+18.\overset{282}{\cancel{455}}+3.052\right)$$

$$\frac{3}{8}(0.2)\left[15.541\right]$$

$$\Rightarrow \boxed{1.827} \text{ Ans}$$

$$\int_4^{5.2} \log x \, dx = x\log x - x$$

$$\Rightarrow (5.2\log 5.2 - 5.2) - (4\log 4 - 4)$$

$$(8.573 - 5.2) - (5.545 - 4)$$

$$\Rightarrow 3.373 - 1.545$$

$$= \boxed{1.828} \text{ Ans}$$

Q Find the value of $\int_{0.2}^{1.4} \sin x \, dx$

$n = 6$ using of three methods and compare of exact value

$$\int_{0.2}^{1.4} \sin x \, dx$$

$$x_n = x_0 + nh$$
$$1.4 = 0.2 + 6h$$

$$\boxed{h = 0.2}$$

$x_0 = 0.2$

$x_1 = x_0 + h$
$= 0.2 + 0.2$
$= 0.4$

$x_2 = x_0 + 2h$
$= 0.2 + 2(0.2)$
$= 0.6$

$x_3 = x_0 + 3h$
$= 0.2 + 3(0.2)$
$\Rightarrow 0.8$

$x_4 = x_0 + 4h$
$= \cancel{1.4}\ 1.0$

$x_5 = x_0 + 5h$
$= 1.2$

$x_6 = 1.4$

$y_0 = \sin x_0 = \sin 0.2 = 0.198$
$y_1 = \sin x_1 = \sin 0.4 = 0.389$
$y_2 = \sin x_2 = \sin 0.6 = 0.\cancel{767}564$
$y_3 = \sin x_3 = \sin 0.8 = 0.717$
$y_4 = \sin x_4 = \sin 1.0 = 0.841$
$y_5 = \sin x_5 = \sin 1.2 = 0.932$
$y_6 = \sin x_6 = \sin 1.4 = 0.985$

(i) Trapezoidal Rule

$$\frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$\frac{0.2}{2}\left[(0.198+0.985)+2(0.389+0.564+0.717+0.841+0.932)\right]$$

$$\frac{0.2}{2}\left[1.183+6.886\right]$$

$$\frac{1.6138}{2} = \boxed{0.8069}$$

(ii) Simpson's 1/3 Rule

$$\frac{1}{3}h\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$\frac{1}{3}(0.2)\left[1.183+4(0.389+0.717+0.932)+2(0.564+0.841)\right]$$

$$\frac{0.2}{3}\left[1.183+8.152+2.81\right]$$

$$= \boxed{0.8096} \text{ Ans}$$

(iii) Simpson's 3/8 Rules

$$\frac{3}{8}h\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$\frac{3}{8}(0.2)\left[1.183+3(0.389+0.564+0.841+0.932)\ 2(0.717)\right]$$

$$\frac{3}{8}(0.2)\left[1.183+8.178+1.434\right]$$

$$\Rightarrow \boxed{0.8096}$$ Ans

Exact value

$$\int_{0.2}^{1.4}\sin x\,dx \cdot \left[-\cos x\,dx\right]_{0.2}^{1.4}$$

$$\Rightarrow -(\cos 1.4 - \cos 0.2)$$

$$\Rightarrow -(0.169 - 0.980)$$

$$= \boxed{0.811}$$

Hence exact value of 0.811 and therefore find differ[illegible] is .1.44

Imp

$$\int_0^6 \frac{dx}{1+x^2} \qquad n = 6$$

$x_n = x_0 + nh$

$6 = 0 + 6h$

$h = 1$

$x_0 = 0$ $\qquad$ $x_3 = 3$

$x_1 = 1$ $\qquad$ $x_4 = 4$

$x_2 = x_0 + 2h$ $\qquad$ $x_5 = 5$

$\Rightarrow 0 + 2(1)$ $\qquad$ $x_6 = 6$

$\Rightarrow 2$

$$y_0 = \frac{1}{1+(x_0)^2} = \frac{1}{1+0} = 1$$

$$y_1 = \frac{1}{1+(x_1)^2} = \frac{1}{1+1} = 0.5$$

$$y_2 = \frac{1}{1+(x_2)^2} = \frac{1}{1+4} = 0.2$$

$$y_3 = \frac{1}{(1+(x_3)^2} = \frac{1}{1+9} = 0.1$$

$$y_4 = \frac{1}{1+(x_4)^2} \quad \frac{1}{1+16} = 0.058$$

$$y_5 = \frac{1}{1+(x_5)^2} \quad \frac{1}{1+25} \Rightarrow 0.038$$

$y_6 = \frac{1}{1+36} = 0.027$

Trapezoidal Rul

$\frac{1}{2}\left[[1+0.027]+2[0.5+0.2+0.1+0.058+0.038]\right]$

$\frac{1}{2}[1.027 + 1.792]$

$\Rightarrow \boxed{1.4095}$

Simpson's Rule 1/3

$\frac{1}{3}(1)\left[(1.027 + 4(0.5+0.1+0.058) + 2(0.2+0.058)\right]$

$\frac{1}{3}(1.027 + 2.552 + 0.516]$

$= \boxed{1.365}$

8

$\int_0^6 \frac{dx}{1+x^2} = (\tan^{-1}x)_0^6 \Rightarrow \tan^{-1}6 - \tan^{-1}0$

38

$\Rightarrow \boxed{1.405}$

Q. Find the value of

Imp $\int_{0.2}^{1.4} (\sin x - \log x + e^x)\, dx \quad n = 6$

by using simpson's Rule 1/3

$x_n \Rightarrow x_0 + nh$

$1.4 = 0.2 + h6$

$h = 0.2$

$x_0 = 0.2 \qquad x_3 = 0.8 \qquad x_6 = 1.4$

$x_1 = 0.4 \qquad x_4 = 1.0$

$x_2 = 0.6 \qquad x_5 = 1.2$

$y_0 = \sin x_0 - \log x_0 + e^{x_0}$

$\Rightarrow \sin 0.2 - \log 0.2 + e^{0.2}$

$= 0.1986 - (-1.609) + 1.221$

$\Rightarrow \boxed{3.0278}$

$y_1 = \sin x_1 - \log x_1 + e^{x_1}$

$\Rightarrow \sin 0.4 - \log 0.4 + e^{0.4}$

$\Rightarrow 0.389 + 0.916 + 1.491$

$\Rightarrow \boxed{2.796}$

$y_2 = \sin x_2 - \log x_2 + e^{x_2}$

$= \sin 0.6 - \log 0.6 + e^{0.6}$

$0.564 + 0.5110 + 1.822$

$= \boxed{2.897}$

$y_3 = \sin x_3 - \log x_3 + e^{x_3}$

$\Rightarrow \sin 0.8 - \log 0.8 + e^{0.8}$

$= 0.717 + 0.223 + 2.225$

$= \boxed{3.165}$

= 1.4

$y_4 = \sin x_4 - \log x_4 + e^{x_4}$

$= \sin 1.0 - \log 1.0 + e^{1.0}$

$0.841 - 0 + 2.718$

$= \boxed{3.559}$

$y_5 = \sin x_5 - \log x_5 + e^{x_5}$

$\Rightarrow \sin 1.2 - \log 1.2 + e^{1.2}$

$0.932 - 0.182 + 3.320$

$\Rightarrow \boxed{4.07}$

2 1

$y_6 = \sin x_6 - \log x_6 + e^{x_6}$

$= \sin 1.4 - \log 1.4 + e^{1.4}$

$0.985 - 0.3316 + 4.055$

$\Rightarrow \boxed{4.704}$

1

Simpson's Rule 1/3

$$\frac{1}{3}(0.2)\left[(3.0288+4.704)+4(2.796+3.165+4.07)+2(2.897+3.559)\right]$$

$$\frac{0.2}{3}\left[7.732+40.124+12.917\right]$$

$$\rightarrow \boxed{4.0510}$$

Exact value

$$\int_{0.2}^{1.4}(\sin x-\log x+e^{x})\,dx$$

$$\left[-\cos x-(x\log x-x)+e^{x}\right]_{0.2}^{1.4}$$

$\cancel{[(\cos 1.4)-(\cos 0.2)]-[1.410}$

$x\log x-x$

$1.4\log 1.4 - 0.2\log 0.2$

$\cancel{0.811} = \cancel{\cos x} \cdot 0.811$

$$-(\cos 1.4-\cos 0.2)$$

$$=(0.169-0.980) \Rightarrow 0.811$$

$x\log x - x$

$(1.4\log 1.4 - 1.4) - (0.2\log 0.2 - 0.2)$

$1.4(\log 1.4 - 1) - 0.2(\log 0.2 - 1)$

$1.4(0.336 - 1) - 0.2(-1.609 - 1)$

$1.4(-0.664) - 0.2(-2.609)$

$-0.9296 + 0.5218$

$= -0.4086$

$e^x = e^{1.4} - e^{0.2}$

$= 4.055 - 1.221$

$= 2.834$

$= 0.811 + 0.4086 + 2.834$

$\Rightarrow \boxed{4.053}$ Ans

$\Rightarrow (\cancel{-\cos 1.4} - 1.4\log 1.4 + 1.4 + e^{1.4}) -$

$(\cancel{-\cos 0.2} - 0.2\log 0.2 + 0.2 + e^{0.2})$

$\Rightarrow (-0.169 - 1.4(0.336) + 1.4 + 4.055) -$

$(-0.980 - 0.2(-1.609) + 0.2 + 1.221)$

$\Rightarrow \boxed{4.053}$

~~Q $\int_{-3}^{3} x^4\,dx$ simpson's 1/3 rule ... exact~~

~~$n = N$~~

~~$x_n = x_0 + nh$~~

~~$3 = -3 + 4h \mid h$~~

~~$h = 1$~~

Q $\int_{-3}^{3} x^4\,dx$ using 7 ordinates
simpson's 1/3

$x_0 \xrightarrow{7} x_n$

$\boxed{n = 6}$

Q $\int_{-3}^{3} x^4\,dx$ using Trapezoidal method exact value

$\boxed{n = 1}$

(7 ordinates / $n = 6$)
both are same

$x_n = x_0 + nh$

$3 = -3 + 6h$

$h = 1$

$x_0 = -3$

$x_1 = x_0 + h$

$\Rightarrow -3 + 1$

$= -2$

$x_2 = x_0 + 2h$

$\Rightarrow \cancel{-3 + 2h}$

$\Rightarrow -1$

$x_3 = 0$ $\quad x_4 = 1$ $\quad x_5 = 2$

$x_6 = 3$

$y_0 = x^4 \Rightarrow (-3)^4 =$
$= 81$

$y_1 = x^4 = (-2) = 16$

$y_2 = 1 \qquad , y_3 = 0$

$y_4 = 1 \qquad y_5 = 16 \qquad y_6 = 81$

$\frac{1}{3}\left[(81+81) + 4(16+0+16) + 2(1+1)\right]$

$\frac{1}{3}\left[162 + 128 + 4\right]$

$\Rightarrow 98$ निय

Exact

$\Rightarrow 2\left(\frac{x^5}{5}\right)_0^3 = 2\left(\frac{243}{5}\right)$

$\Rightarrow \boxed{97.2}$

x variable coefficient -1 than shift the 1st equation

13 oct

Unit - 4

Imp

Solution of linear Equation

Gauss's Elimination Method — Gauss-seidel Method

1) Gauss's Elimination Method :-

Suppose three variables & three equation general from of linear equation is :-

$a_{11}x + a_{12}y + a_{13}z = a_{14}$ — (1)

$a_{21}x + a_{22}y + a_{23}z = a_{24}$ — (2)

$a_{31}x + a_{32}y + a_{33}z = a_{34}$ — (3)

a_{ij} is → cofficient of variable

x, y, z → is variable

Step 1) Eliminate x from equation 2nd & 3rd with help of equation (1)

$$\left(a_{21} - \frac{a_{21}}{a_{11}} a_{11}\right)x + \left(a_{22} - \frac{a_{21}}{a_{11}} a_{12}\right)y + \left(a_{23} - \frac{a_{21}}{a_{11}} a_{13}\right)z = a_{24} - \frac{a_{21}}{a_{11}} a_{14}$$

$$\boxed{R_3 \to R_3 - \frac{a_{31}}{a_{11}} R_1}$$

$$\left(a_{31} - \frac{a_{31}}{a_{11}} a_{11}\right) x + \left(a_{32} - \frac{a_{31}}{a_{11}} a_{12}\right) . y +$$

$$\left(a_{33} - \frac{a_{31}}{a_{11}} a_{13}\right) z = a_{34} - \frac{a_{31}}{a_{11}} a_{14}$$

$$a_{11} x + a_{12} y + a_{13} z = a_{14} \quad — (1')$$

$$a_{22} y + a_{23} z = a_{24} \quad — (2')$$

$$a_{32} y + a_{33} z = a_{34} \quad — (3')$$

step 2) eliminate y from equation 3^{rd} with help of (II)

$$a_{11} x + a_{12} y + a_{13} z = a_{14} \quad — (1)$$

$$a_{22} y + a_{23} z = a_{24} \quad — (2)$$

$$\boxed{R_3 \to R_3 - \frac{a_{32}}{a_{22}} R_2}$$

$$\left(a_{32} - \frac{a_{32}}{a_{22}} a_{22}\right) y + \left(a_{33} - \frac{a_{32}}{a_{22}} a_{23}\right) z =$$

$$a_{34} - \frac{a_{32}}{a_{22}} a_{24}$$

$$a_{33} z = a_{34} \quad — (3)$$

* z - (iii) y - (ii) x - (i) are value to be calculated

Q For the following linear eq^n -

$$x + 2y = 5$$
$$3x - y = 1$$

by using Gauss's Eliminated Method

$$x + 2y = 5 \times 3$$
$$3x - y = 1 \times 1$$

$$\cancel{3x} + 6y = 15$$
$$\cancel{3x} - y = 1$$

$$7y = 14$$
$$y = 2 \quad , x = 1$$

$$x + 2y = 5 \quad \text{— (1)}$$
$$3x - y = 1 \quad \text{— (2)}$$

(Step 1)

$$x + 2y = 5$$

$$(3-3)x + (-1-3\cdot 2)y = 1-5\cdot 3$$

$$0 \; -7y = -14$$

$$\boxed{y = 2}$$

$$\boxed{x = 1}$$

Q Solve the following linear ~~eq~~ eq by using Gauss's Elimination method

$$x + 4y = 10 \quad - ①$$
$$6x + 5y \quad 6 \quad - ②$$

(Step 1) $x + 4y = 1$

$$(6-6)x + (5-6\cdot 4)y = 10 - 1\cdot 6$$

$$0x + (-19) = -4$$

$$-19y = -5$$

Q. Solve the following eq^n

$$3x + y - z = 3$$
$$x + 2y + z = 4$$
$$4x - y + 3z = 6$$

Sol Change 1^st & 2^nd

$$x + 2y + z = 4 \quad \text{--- } (1)$$
$$3x + y - z = 3 \quad \text{--- } (2)$$
$$4x - y + 3z = 6 \quad \text{--- } (3)$$

Step 1) Eliminate x from eq 2^nd & 3^rd with the help of eq (1)^st

$$x + 2y + z = 4 \quad \text{--- } (1)$$

$$R_2 \to R_2 - \tfrac{3}{1} R_1$$

$$\left(\cancel{3 - \tfrac{3}{1} \cdot 1}\right) x + \left(1 - \tfrac{3}{1} \cdot 2\right) y + \left(-1 - \tfrac{3}{1} \cdot 1\right) z = 3 - \tfrac{3}{1} \cdot 4$$

$$\Rightarrow \quad -5y - 4z = -9$$

$R_3 \to R_3 - 4/1\, R_1$

$\cancel{(4 - \frac{4}{1} \cdot 1)x} + (-1 - \frac{4}{1} \cdot 2)y +$

$(3 - \frac{4}{1} \cdot 1)z = 6 - \frac{4}{1} \cdot 4$

$-9y - z = -\cancel{18}10$

3rd

$x + 2y + z = 4$
$-5y - 4y = -9$
$-9y - z = -10$

step1)

$x + 2y + z = 4$ — (1)
$5y + 4z = 9$ — (2)
$9y + z = 10$ — (3)

Step2) Eliminate y from (3) within (2)

$x + 2y + z = 4$
$5y + 4z = 9$

$\cancel{(9 - \frac{9}{5} \cdot 5)y} + (5 - \frac{9}{5} \cdot 1)z =$

$10 - \frac{9}{5} \cdot 9$

Date ______
Page ______

$$x + 2y + z = 4 \quad \text{—} \ (1)$$

$$5y + 4.z = 9 \quad \text{—} \ (2)$$

$$\cancel{-21} - \frac{31}{5}z = \frac{-31}{5} \quad \text{—} \ (3)$$

from eq (3)

$$z = 1$$

from eq (2) put $z = 1$

$$5y + 4z = 9$$

$$5y = 5$$

$$y = 1$$

from eq (1) put z & y value

$$x + 2y + z = 4$$

$$x = 4 - 2 - 1$$

$$\boxed{x = 1}$$

$$\boxed{x = 1,\ y = 1,\ z = 1}$$

~~3x + 2y + 2z~~

$$3x + y - z = 3$$

$$3(1) + 1 - 1 = 3$$

$$3 + 1 - 1 = 3$$

$$0 = 0$$

Q. Solve the following linear eqn.

$$x + 2y + 3z = 5 \quad — (1)$$
$$2x + y + 4z = 5 \quad — (2)$$
$$5x - y + 2z = 1 \quad — (3)$$

Step 1) Eliminate x from eq 2^{nd} & 3^{rd} with the help of eq 1^{st}

$$x + 2y + 3z = 5$$

$$R_2 \to R_2 - \frac{2}{1} R_1$$

$$\left(2 - \frac{2}{1}\cdot 1\right)x + \left(1 - \frac{2}{1}\cdot 2\right)y + \left(4 - \frac{2}{1}\cdot 3\right)z = 5 - \frac{2}{1}\cdot 5$$

$$\boxed{-3y - 2z = -5}$$

$R_3 \rightarrow R_3 - \frac{5}{1} R_1$

$\left(5 - \frac{5}{1}1\right)x + \left(-1 - \frac{5}{1}2\right)y +$

$\left(2 - \frac{5}{1}3\right)z = 1 - \frac{5}{1}5$

$-11y - 13z = -24$

$x + 2y + 3z = 5$
$-3y - 2z = -5$
$-11y - 13z = -24$

(Step 1)

$x + 2y + 3z = 5$ — (1)

$3y + 2z = 5$ — (2)

$11y + 13z = 24$ — (3)

(Step 2) Eliminate y from (3) with in (2)

$x + 2y + 3z = 5$
$3y + 2z = 5$

$-1 - 10 \quad 13 - \frac{22}{3} \quad \frac{39 - 22}{3}$

Date
Page

$\Rightarrow \left(11 - \cancel{\frac{11}{3} \cdot 3}\right) y + \left(13 - \frac{11}{3} 2\right) z$

$\Rightarrow 24 - \frac{11}{3} \cdot 5$

$\Rightarrow \frac{17}{3} z = \frac{17}{3}$

$\Rightarrow x + 2y + 3z = 5 \quad - (1)$

$3y + 2z = 5 \quad - (2)$

$\frac{17}{3} z = \frac{17}{3} \quad - (3)$

from eq = (3)

$z = 1$

from eq (2)

$3y = 5 - 2$

$3y = 3$

$y = 1$

from eq (1)

$x + 2y + 3z = 5$

$x + 2(1) + 3(1) = 5$

$x = 5 - 3 - 2$

$x = 0$

$x = 0, \; y = 1, \; z = 1$

Q Solve the following linear eqn

$$2x + y + 2z = 9$$
$$x + 4y - z = 2$$
$$5x + y - 3z = -3$$

change (1) to (2)

$$x + 4y - z = 2 \quad \text{—} \ (1)$$
$$2x + y + 2z = 9 \quad \text{—} \ (2)$$
$$5x + y - 3z = -3 \quad \text{—} \ (3)$$

Step 1)

$$R_2 \to R_2 - \frac{2}{1} R_1$$

$$\cancel{\left(2 - \frac{2}{1} \cdot 1\right)} x + \left(1 - \frac{2}{1} \cdot 4\right) y + \left(2 - \frac{2}{1}(-1)\right) z = \left(9 - \frac{2}{1} \cdot 2\right)$$

$$-7y + 4z = 5$$

$$R_3 \to R_3 - \frac{5}{1} R_1$$

$$\cancel{\left(5 - \frac{5}{1} \cdot 1\right)} x + \left(1 - \frac{5}{1} \cdot 4\right) y + \left(-3 - \frac{5}{1}(-1)\right) z = \left(-3 - \frac{5}{1} \cdot 2\right)$$

$-19y + 2z = -13$

$x + 4y - z = 2$ — 1

$-7y + 4z = 5$ — 2

$-19y + 2z = -13$ — 3

Step.) Eliminate y from 3 with 1x 2 $(19/7)$

$$\left(-19 - \cancel{\tfrac{19}{7}(-7)}\right)y + \left(2 - \tfrac{19}{7}4\right)z$$

$$\Rightarrow -13 - \frac{19}{7}5$$

$$\Rightarrow \frac{-62z}{7} = \frac{-186}{7}$$

$$z = \frac{-186}{\cancel{7}} \times \frac{\cancel{7}}{62}$$

$$z \Rightarrow \boxed{3}$$

from 2 y

from (1) x

$$x + 4y - z = 2$$

$$x + 4(1) - 3 = 2$$

$$x = 2 + 3 - 4$$

$$\boxed{x = 1}$$

$$\boxed{x = 1,\ y = 1,\ z = 3}$$

22/oct

2) Gauss's Seidel Method:-

~~iterationequation~~ method (step) is the general form of linear equation for three variables.

$$a_{11}x + a_{12}y + a_{13}z = a_{14} \quad \text{— (I)}$$

$$a_{21}x + a_{22}y + a_{23}z = a_{24} \quad \text{— (II)}$$

$$a_{31}x + a_{32}y + a_{33}z = a_{34} \quad \text{— (III)}$$

x do calculated form — (I)

y " " — (II)

z " " — (III)

iteration	x	y	z
1	–	–	–
2	–	–	–

x from eq (I)
y " " (II)
z " " (III)

in eq (I) y & z will be 0 x than calculated x from eq (I) after this value of x eq (I) and z will be 0 than calculate y from eq (II) & after value of x & y from (I) eq^n calculate the value of z from eq^n (III)

Q) Solve the following linear eq^n by using Gauss's Seidel Method.

$$2x + y = 3 \quad — (I)$$
$$x + 3y = 4 \quad — (II)$$

$2x + y = 3$	$x + 3y = 4$
$x = \frac{3-y}{2}$	$y = \frac{4-x}{3}$

$x = \frac{3-0}{2}$ $y = \frac{4-1.5}{3}$, $\frac{3-0.833}{2}$

$y = \frac{4-1.083}{3}$

iteration	x	y
1	1.5	0.833
2	1.083	0.972
3	1.014	0.995
4	1.003	0.999
5	1.001	0.999

last eqⁿ nearly to same i.e.
$x = 1.001$ & $y = 0.999$ Ans

Q) Solve the following linear eqⁿ by G.S.M.

$$x + 3y + z = 8$$
$$3x + y + 2z = 7$$
$$2x - y + 4z = 4$$

$$3x + y + 2z = 7 \quad \text{---(i)}$$
$$x + 3y + z = 8 \quad \text{---(ii)}$$
$$2x - y + 4z = 4 \quad \text{---(iii)}$$

$3x+y+2z=7$	$x+3y+z=8$	$2x-y+4z=4$
$x = \frac{7-y-2z}{3}$	$y = \frac{8-x-z}{3}$	$z = \frac{4-2x+y}{4}$

iteration	x	y	z
1	2.333	1.889	0.305
2	1.500	2.065	0.766
3	1.134	2.033	0.941
4	1.028	2.010	0.988
5	1.004	2.002	0.998
6	1.001	2.000	0.999

$$\left.\begin{aligned} x &= 1.001 \\ y &= 2.000 \\ z &= 0.999 \end{aligned}\right\} \text{Ans}$$