14/Aug/18

Unit - 1.

Roots of Eqn.

There are 3 methods to find the root of equation :-

① Bisection method.

② False Position method

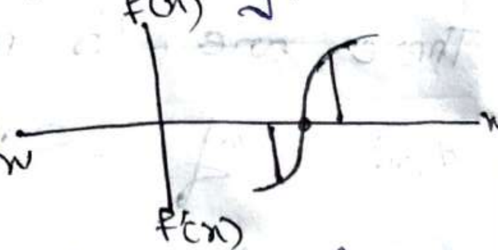
③ N-R method (Newton - Raphson)

To find any root of the fn. $f(x)$ by Iteration Method (steps) if the value of two equations is nearly to same or equal then find the root (x) and the value of x is last value of equations.

Bisection Method

1) Initially suppose any two values x_0 & x_1.

2) Now find the value of $f(x_0)$ & $f(x_1)$, but the condition is $f(x_0)$ and $f(x_1)$ has the opposite sign. (+ve and -ve).



3) Now find the midpoint by using formula $x_2 = \frac{x_0 + x_1}{2}$.

and then calculate the value of $f(x_2)$.

4) Check the nature of $f(x_2)$.

5) If $f(x_2)$ is negative then value $\boxed{x_0 \leftarrow x_2.}$

If $f(x_2)$ is positive then value replace by $\boxed{x_1 \leftarrow x_2}$

6) Again find the midpoint

$$x_2 = \frac{x_0 + x_1}{2}.$$

again find $f(x_2)$ ---- continues. until the value of last two equations x_2 are ~~aee~~ nearly to equal.

Now the root is last value of x_2.

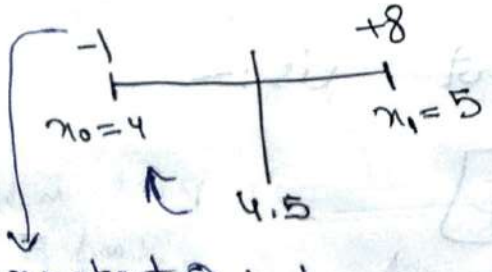
Q) Find the root of equation of the fn $f(x) = x^2 - 17$. by Bisection method and answer should be upto 3 decimal places.

Sol). Let us suppose that x_0 and x_1 be 4 and 5 respectively.

$x_0 = 4 \qquad f(x) = 4^2 - 17 = \boxed{-1}$

$x_1 = 5 \qquad f(x) = x^2 - 17 = 5^2 - 17$

$= \boxed{+8}$



near about 0 is -1
So the answer
is below 4.5.

$$x_2 = \frac{x_0 + x_1}{2}$$

$$x_2 = \frac{4+5}{2}$$

$$\boxed{x_2 = 4.5}$$

$$f(x_2) = x^2 - 17$$
$$= (4.5)^2 - 17$$
$$= 3.25 \text{ i.e } +ve.$$

+ will be replace by + and vice versa

No. of step	Change Variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$
1.	—	4	5	−	+	4.5	±
2.	$x_1 \leftarrow x_2$	4	4.5	−	+	4.25	+
3.	$x_1 \leftarrow x_2$	4	4.25	−	+	4.125	+
4.	$x_1 \leftarrow x_2$.	4	4.125	−	+	4.0625	−
5.	$x_0 \leftarrow x_2$ (because − will be replace by negative)	4.0625	4.125	−	+	4.093	−
6.	$x_0 \leftarrow x_2$.	4.093	4.125	−	+	difference 0.101 4.109	

$\Rightarrow$ 2 step :- $f(x_2) = (4.25)^2 - 17 = +ve.$

The value of last two equations are nearly to same and hence the root is :-

$$\boxed{x = 4.109} \Longrightarrow \text{point in b/w 4 and 5}$$

Ans

Q). Solve the following linear eqn.
$f(x) = \log x - \cos x$ by using bisection method. upto 3 decimal places.

Sol) when $x_0 = 1.1$

$f(x) = \log x - \cos x$
$= 0.0953 - 0.453$
$f(x) = -0.359$ i.e $\boxed{-ve}$

$x_1 = 1.4$

$f(x) = 0.336 - 0.169$
$= \boxed{+}$ ~~0.166~~ 0.176 i.e $\boxed{+ve}$

$x_0 = 1.3$ and $x_1 = 1.4$

$f(x_0) = -ve = .005$
$f(x_1) = +ve = .16$

$$x_2 = \frac{x_0 + x_1}{2} = 1.35$$

$f(x_2) = \log 1.35 - \cos 1.35$
$= 0.3001 - 0.219$
$f(x_2) = +ve = 0.0811$

No. of step	change variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$
1.	—	1.3	1.4	—	+	1.35	+
2.	$x_1 \leftarrow x_2$	1.3	1.35	—	+	1.325	+
3.	$x_1 \leftarrow x_2$	1.3	1.325	—	+	1.3125	+
4.	$x_1 \leftarrow x_2$	1.3	1.3125	—	+	1.30625	+

The value of $f(x_2)$ is nearly to zero in 4th equation.

Hence the root is $\boxed{x = 1.30625}$

Answer is $\boxed{x = 1.306}$

Q) Solve the linear equation $f(x) = x^3 - 9x - 1$. The root lie b/w $(3,4)$

Sol) $x_0 = 3$ | $x_1 = 4$.

$f(x_0) = -ve = -1$

$f(x_1) = +ve. = 27$

$$x_2 = \frac{x_0 + x_1}{2} = \frac{3+4}{2} = 3.5$$

$$f(x_2) = (3.5)^3 - 9 \times 3.5 - 1$$

$$= 42.8 - 30.5$$

$$= 12.3$$

No. of step	change variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$
1.	—	3	4	−	+	3.5	+
2.	$x_1 \leftarrow x_2$	3	3.5	−	+	3.25	+
3.	$x_1 \leftarrow x_2$	3	3.25	−	+	3.125	+
4.	$x_1 \leftarrow x_2$	3	3.125	−	+	3.0625	+
5.	$x_1 \leftarrow x_2$	3	3.0625	+	+	3.031	+

False Position Method

$f(x) = x / \cos x / \log / e^{x}$.

(1) x_0 and x_1

(2) $f(x_0)$ & $f(x_1)$

or

$f(x_0) f(x_1) < 0$

(3) $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}.$$

(3.1) $f(x_2) < ^{+}_{-}$

Suppose two values x_0 and x_1 then calculate the value of $f(x_0)$ and $f(x_1)$, the value of $f(x_0)$ and $f(x_1)$ have the opposite sign i.e $f(x_0) f(x_1) < 0$

Now find the mid point by using formula $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

then calculate the value of $f(x_2)$ and check the nature of $f(x_2)$.

(i) if $f(x_2)$ is negative then x_0 replace by x_2 and $f(x_0) \leftarrow f(x_2)$

(ii) if $f(x_2)$ is positive then $x_1 \leftarrow x_2$

$$f(x_1) \leftarrow f(x_2).$$

Again find the value of x_2 and then find the nature of $f(x_2)$. until the value of $f(x_2)$ is nearly to 0. ($f(x_2) \simeq .00$ No.)

Q) Find the root of eqn. of the fn. $f(x) = x\log_{10} x - 1.2$. by ~~using~~ using false position ~~bisection method~~, the root lie b/w interval (2, 3).

$\log_{10} 1 = 0$
$\log_1 1$ = Undefined
$\log_3 3 = 1 \frac{\log_e 3}{\log_e 3}$

Sol) $x_0 = 2 \qquad x_1 = 3.$

$$f(x_0) = 2\log_{10} 2 - 1.2.$$
$$= 2 \cdot 0.301 - 1.2.$$
$$= 0.602 - 1.2$$
$$= -ve. \ (-0.597).$$

$$f(x_1) = 3\log_{10} 3 - 1.2$$
$$= 3 \times 0.47 - 1.2$$
$$= 1.43 - 1.2$$
$$= +ve \ \boxed{0.231}$$

$$x_2 = \frac{x_0 + x_1}{2} = \frac{2+3}{2} = \frac{5}{2}$$

$$= \boxed{2.5}$$

$$f(x_2) = x \log_{10} x - 1.2$$

$$= 2.5 \log_{10} 2.5 - 1.2$$

$$= \boxed{-0.205}$$

No. of

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{2 \times 0.231 - 3(-0.597)}{0.231 - (-0.597)}$$

$$x_2 = \frac{0.462 + 1.791}{0.828}$$

$$\boxed{x_2 = 2.721}$$

$$f(x_2) = x \log_{10} 2.721 - 1.2$$

$$= 2 \cdot \log_{10} 2.721 - 1.2$$

$$f(x_2) = \boxed{-0.017}$$

No. of Steps	
1.	
2.	

$x_2 =$

$x_2 =$

The val

in

root

No. of Steps	Change of Variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$
1.	—	2	3	-0.597	+0.231	2.721	-0.017
2.	$x_0 \leftarrow x_2$	2.721	3	-0.017	0.0231		

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{2.721(0.0231) - 3(-0.017)}{0.0231 + 0.017}$$

~~x_2~~

The value of $f(x_2)$ is nearly to zero in second equation. Hence the root is $\boxed{x = 2.740}$.

(I) if $f(x_2)$ is negative and $f(x_0) \leftarrow$ ... replace by x_2

(II) if $f(x_2)$ is positive then $x_1 \leftarrow x_2$

$f(x_1) \leftarrow f(x_2)$.

Again find the value of x_2 and then find the nature of $f(x_2)$.
until the value of $f(x_2)$ is nearly to 0. ($f(x_2) \simeq .00\ldots$)

Q) Find the root of eqn. of the fn. $f(x) = x\log_{10} x - 1.2$ by using false position ~~bisection method~~ the root lie b/w interval $(2,3)$.

$$\boxed{\begin{aligned} &\log_{10} 1 = 0 \\ &\log_1 1 = \text{Undefined} \\ &\log_3 3 = 1 = \frac{\log_e 3}{\log_e 3} \end{aligned}}$$

Sol) $x_0 = 2 \qquad x_1 = 3$.

$$\begin{aligned} f(x_0) &= 2\log_{10} 2 - 1.2 \\ &= 2 \cdot 0.301 - 1.2 \\ &= 0.602 - 1.2 \\ &= -\text{ve}. \ (-0.597) \end{aligned}$$

$$\begin{aligned} f(x_1) &= 3\log_{10} 3 - 1.2 \\ &= 3 \times 0.4\ldots \end{aligned}$$

No. of Steps	Change of Variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$
1.	—	2	3	-0.597	+0.231	2.721	-0.017
2.	$x_0 \leftarrow x_2$	2.721	3	-0.017	0.0231		

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{2.721(0.0231) - 3(-0.017)}{0.0231 + 0.017}$$

The value of $f(x_2)$ is nearly to zero in second equation. Hence the root is $\boxed{x = 2.740}$.

Q) Find the root of equation of the $f(x) = \cos x - x e^{x}$. by using false position method. The root lie b/w $(0,1)$.

Sol) $f(x) = \cos x - x e^{x}$

$x_0 = 0 \qquad x_1 = 1$

$f(x_0) = \cos 0 - 0e^{0} = +1$

$f(x_1) = \cos 1 - 1e^{1} = 0.540 - 2.7818 = \boxed{-2.\cancel{2418}\,178}$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\cancel{x_2 = \frac{0 \times 1 - (-2.178) \times 1}{-2.178 - 1}}$$

$$x_2 = \frac{0(-2.718) - 1(1)}{(2.718) - 1}$$

$$x_2 = \frac{-1}{3.178} = \boxed{-0.314}$$

$f(x_2) =$

$=$

of the
sing False
ie b/w

$+1$
$0.540 -$
2.7818
$\boxed{2.\cancel{2418}\,178}$

(1)

$$f(x_2) = \cos(-0.314) - (-0.314) \times e^{-0.314}$$
$$= 0.681$$

Q) $f(x) = \cos x - xe^{x}$. $(0,1)$.

Sol) $f(x) = \cos x - xe^{x}$.

$x_0 = 0 \qquad x_1 = 1$

$f(x_0) = +1$

$f(x_1) = -2.178$.

$$x_2 = \frac{x_0 (f x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\boxed{x_2 = \frac{-1}{3.178}}$$

$f(x_2) = \cos x - xe^{x}$.

$= 0.521$.

No. of Step	Change of Variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	f(
1.	—	0	1	1	-2.178	0.314	
2.	$x_0 \leftarrow x_2$ $f(x_0) \leftarrow f(x_1)$.	.314	1	.521	-2.178	.44	
3.	$x_0 \leftarrow x_2$. $f(x_0) \leftarrow f(x_1)$	.446	1	.205	-2.178	0.	
4.	$x_0 \leftarrow x_2$.	.493	1	.093	-2.178	0	
5.	$x_0 \leftarrow x_1$	.509	1	.262	-2.178		

6. $x_0 \leftarrow x_1$ 0.514 1 0.11 −2.178 0.516.

.00

The value of $f(x_2)$ is nearly to zero sixth equation and hence the root is $\boxed{x = .516}$

Q) $f(x) = x^2 - x - 1$ by using bisection & false position method.

Sol) Let us suppose that x_0 and x_1 be 1 and 2 respectively.

$x_0 = 1$ $f(x_0) = 1^2 - 1 - 1 \Rightarrow -1$ −ve

$x_1 = 2$ $f(x_1) = 4 - 2 - 1 \Rightarrow 1$ +ve.

$$x_2 = \frac{1+2}{2} = \frac{3}{2} = 1.5$$

$$f(x_2) = (1.5)^2 - (1.5) - 1$$

$$= 2.25 - 2.5$$

0.521.

$$= -0.25$$

0.205.

No. of Steps	Change of Variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$
1.	−	−1	1	−	+	1.5	−
2.	$x_0 \leftarrow x_2$.	1.5	1	−	+	1.25	
3.	$x_0 \leftarrow x_2$.	1.25	1	−	+	1.125	
4.	$x_0 \leftarrow x_2$.	1.125	1	−	+	1.06	

0.073

0.262.

4 0.11

Brisection Method :-

Taking $x_0 = 1$, $x_1 = 2$.

$f(x_0) = -1$ $\quad$ $f(x_1) = +1$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{1 \times (+1) - (+1)(-1)}{1 - (-1)}$$

$$x_2 = \frac{1 + 1}{2} = \frac{2}{2} = \boxed{1}$$

$$f(x_2) = 1^2 - 1 - 1 = 1 - 2 = \boxed{-1}$$

No. of Steps.	Change of Variable	x_0	x_1	$f(x_0)$	$f(x_1)$	x_2	$f(x_2)$

Imp: ~~Metho~~

(iii) Newtons - Raphson Method.

$f(x) = x \,/\, \cos x \,/\, \log x \,/\, e^{x} - \cos y$

interval $(1, 2)$

any one.

→ value of x_0. (Take only one value x_0 or x_1)

diffn. the fn. which are given w.r.t x

cal. iteration $\left[x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}\right.$.

$[x_2 = x_1 - f(x_1)/f'(x_1)]$

$[x_3 = x_2 - f(x_2)/f'(x_2)]$

Two value same then it is ans.

General eqn. of N-R method is:-

$$x_{n+1} = x_n = \frac{f(x_n)}{f'(x_n)}$$

where $n = 0, 1, 2, 3 \ldots$

- Suppose a single value in the interval (a,b)

(i) Find the root of eqn. of the fn.

$$f(x) = x^3 - 2x - 5$$

by using N-R method upto 3 decimal places.

$$f(x) = x^3 - 2x - 5$$
$$f(0) = 0 - 0 - 5 = -5$$
$$f(1) = 1 - 2 - 5 = -6$$
$$f(2) = 8 - 4 - 5 = -1$$
$$f(3) = 27 - 6 - 5 = 16.$$

interval $(2,3)$

Suppose $x_0 = 2.$

$$f'(x) = 3x^2 - 2.$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{8-4-5}{12-2}$$

$$= 2 - \frac{(-1)}{10}$$

$$x_1 = \frac{20+1}{10} = \frac{21}{10} = 2.1$$

$x_2 =$

=

$x_3 =$

=

=

$x_0 =$
$x_1 =$
$x_2 =$
$x_3 =$

The valu

Same i

~~ist~~ ite

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.1 - \frac{0.061}{11.23} = 2.094$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.094 - \frac{(-0.008)}{11.154}$$

$$= 2.095.$$

$$\left.\begin{aligned} x_0 &= 2 \\ x_1 &= 2.1 \\ x_2 &= 2.094 \\ x_3 &= 2.095 \end{aligned}\right\} \text{0.001 nearly Same.}$$

The value of x are nearly to Same in second & third ~~ist~~ iteration, the root is

$$\boxed{x = 2.095} \ \#.$$

Q) Imp. Find the root of eqn. of the fn :- $f(x) = x\log_{10} x - 1.2$ root lie b/w $(2,3)$.

$$f(x) = x\log_{10} x - 1.2$$

Suppose $x_0 = 3$.

$$f'(x) = x \cdot \frac{d}{dx}\log_{10} x + \log_{10} x \frac{d}{dx}(x)$$

$$= x \cdot \frac{1}{x} \cdot \log_{10} e + \log_{10} x.$$

$$f'(x) = \log_{10} e + \log_{10} x = \log_{10} 2.718 + \log_{10} x$$

$$= 0.4342 + \log_{10} x.$$

$e = 2.718$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 3 - \frac{0.2313}{0.9113}$$

$$\boxed{x_1 = 2.746.}$$

$x_2 =$

$=$

$=$

$x_0 =$

$x_1 =$

$x_2 =$

The value

Same in

the roo

Q) $f(x) =$

Raphson

Answer

$\cancel{f(x)} =$

$\cancel{f(0)} =$

$f(0) =$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.746 - \frac{0.0046}{0.8729}$$

$$= 2.741$$

$$x_0 = 3$$

$$\left.\begin{array}{l} x_1 = 2.746 \\ x_2 = 0.741 \end{array}\right\} 0.005$$

The value of x are nearly to same in I^{st} & II^{nd} iteration, the root is :-

$$\boxed{x = 2.741} \; \#$$

Q) $f(x) = e^{-x} - \sin x$. by Newton-Raphson method root b/w $(0,1)$. Answer upto 3 decimal places.

$$\cancel{f(x)} = \cancel{e^{-x}} - \sin \cancel{x}$$

$$\cancel{f(0)} = \cancel{e^{-0}} - \cancel{\sin x}$$

$$f(0) = \cancel{1} - \cancel{\sin 0}$$

$$= 0$$

Answer is in b/w 0 and 1 i.e interval

$f(0) = 1$

$f(1) = -.48.$ 11.47

$$f(x) = e^{-x} - \sin x.$$

$$= \cancel{} \boxed{-e^{-x} - \cos x.}$$

Suppose $x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$= 1 - \frac{+.474}{+.908}$$

$$= 1 - 0.522.$$

$$= \boxed{0.478}$$

$x_0 = 1$
$x_1 = 0.478$
$x_2 = 0.583$
$x_3 = 0.5884.$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.478 + \frac{0.159}{1.508}$$

$$= 0.478 + 0.10$$

$$= \boxed{0.583}$$

$$\begin{array}{r} 0.5884 \\ 0.583 \\ \hline 0.0054 \end{array}$$

$x_3 =$

$= .$

$x_3. =$

Last two

0.5884,

Hence

Q) Find

$x^2 + 9$

falseposition

$f(0)$

$f(1)$

Suppose

$x_1 =$

$x_1 =$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.583 + \frac{0.0076}{1.393}$$

$$x_3 = \boxed{0.5884}$$

Last two equations i.e 0.583 and 0.5884, are nearly to same. Hence the root is $\boxed{0.5884}$

Q) Find the root of equation $f(x) = x^2 + 9x - 1$, using bisection, falseposition and n-r method.

$$f(0) = 0 + 0 - 1 = \boxed{-1}$$

$$f(1) = 1 + 9 - 1 = 10 - 1 = \boxed{9}$$

Suppose $x_0 = 0$. $\quad f'(x) = 2x + 9$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 - \frac{(-1)}{9} = 0.111$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.111 - .0.001$$

$$x_2 = .109$$

$$x_3 = .109.$$

The value of x_2 and x_3 are nearly same. Hence the root is $\boxed{.109}$.

Q) $f(x) = \log x - \cos x$.

~~Interval (0,1)~~ Interval $(1,2)$

(Answer is in b/w 1 and 2).

$$f(1) = \log 1 - \cos 1$$
$$= 0 - .540 = -.540$$

$$f(2) = \log_e 2 - \cos 2.$$
$$= 0.693 - (-.416)$$
$$= +\text{ve}. \ \boxed{1.109} \qquad (1,2).$$

$$f'(x) = \frac{1}{x} + \sin x.$$

$$\boxed{x_0 = 1}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 1 - \frac{(-.540)}{1.84}.$$
$$= 1 + \frac{.540}{1.84} = \boxed{1.293}.$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

UNIT-2 Finite - Differences (Foward & Backword)

$x \rightarrow x_0, x_1, x_2, x_3 \ldots$

$y \rightarrow y_0, y_1, y_2, y_3 \ldots$

$\Delta y \rightarrow$ First $\Delta^2 y \rightarrow$ Second $\Delta^3 y \rightarrow$ Third and so on ----

$\Delta =$ next value − previous value.

$\Delta y_0 = y_1 - y_0$ $\Delta y_3 = y_4 - y_3$

$\Delta y_4 = y_5 - y_4$

$\Delta y_n \rightarrow y_{n+1} - y_n.$

First value is always y_0

$$\boxed{\Delta^2 y_n \rightarrow \Delta y_{n+1} - \Delta y_n.}$$

always first value is y_0 and so on.

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	1	y_0 13				
x_1	3	y_1 19	Δy_0 6			
x_2	5	y_2 21		-4 $\Delta^2 y_0$		
x_3	7		Δy_1 2		7 $\Delta^3 y_0$	
		y_3 26	Δy_2 5	3 $\Delta^2 y_1$		-4 $\Delta^4 y_0$
x_4.	9.	y_4. 37.	Δy_3 11	6 $\Delta^2 y_2$	3. $\Delta^3 y_1$	

Q) Construct a forward difference table of the following condition:-

x	0	1	2	3	4
y	3	6	11	18	27

and then find the value Δy_1, $\Delta^2 y_0$, $\Delta^3 y_1$, y_3.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	3				
		3			
1	6		2		
		5		0	
2	11		2		0
		7		0	
3	18		2		
		9			
4	27				

$\Delta y_1 = 5$ $\qquad \Delta^3 y_1 = 0$

$\Delta^2 y_0 = 2.$ $\qquad y_3 = 18.$

If $\Delta^3 y_2$ is given which is not in table then put dash (–) or cannot find.

$\Delta^3 y_2 = -$

(Inside the interval)

Newton forward Interpolation Method (Equal method):-

If x is $x_0, x_1, x_2, x_3 + - - - -$ and corresponding value y is $y_0, y_1, y_2, y_3 - - - - - - y_n$ then the general Newton forward Interpolation formula is :-

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + - - -$$

where $p = \dfrac{x_n - x_0}{h}$

x_n → point to be calculated.
x_0 → first value of x column.
h → Equal Interval.

$$\boxed{x_n = x_0 + ph.}$$

P is strictly greater than zero.

$\boxed{P>0}$ always.

Q.

x	19
y	1

value o

newton's

Method.

Sol)

	x	y
x_0	1951	y_0 13
x_1	1961	y_1 19
x_2	1971	y_2 2
x_3	1981	y_3 3

$p = \dfrac{x_n - x}{h}$

$= \dfrac{3}{10}$

$f(x) = y_0 + p\Delta y$

$= 13 + (0.5)(6$

$\dfrac{(0.5)(0.5-1)}{2 \times}$

$= 13 + 3 - 0$

Q.

x	1951	1961	1971	1981
y	13	19	27	31.

find the value of y at $x = 1956$ by using newton's forward Interpolation Method.

Sol)

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	1951	y_0 13			
			6		
x_1	1961	y_1 19		2	
			8		-6.
x_2	1971	y_2 27		-4	
			4.		
x_3	1981	y_3 31			

-0.5
-1.5

$$p = \frac{x_n - x_0}{h} = \frac{1956 - 1951}{10}$$

$$= \frac{3}{10} = \boxed{0.5}$$

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 13 + (0.5)(6) + \frac{(0.5)(0.5-1)}{2}(2) + \frac{(0.5)(0.5-1)(0.5-2)}{\cancel{3} \times \cancel{2}} \times \cancel{(-6)}$$

$$= 13 + 3 - \frac{0.5}{2} + (-0.375)$$

$\Rightarrow 13 + 3 - 0.25 + 0.375$

$\Rightarrow \boxed{15.375} \Rightarrow$ Ans is in b/w 13 and 19 because 1956 is in b/w 1951 and 1961.

if $x = 1965$.

$$P = \frac{x_n - x_0}{h} = \frac{1965 - 1951}{10}$$

$$P = \frac{14}{10} = \boxed{1.4}.$$

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$f(x) = 13 + 1.4(6) + \frac{(1.4)(1.4-1)}{2} \cdot 2 + \frac{(1.4)(1.4-1)(1.4-2)}{6} \times -6$$

$$f(x) = 13 + 8.4 + 0.56 + 0.336$$
$$= 22.296$$

Q) Conside…

x	0…
y	

cubic …

foward …

and a…

	x	y	
x_0	0	1	y_0
x_1	1	0	y_1
x_2	2	1	y_2
x_3	3	10	y_3

$P = …$

$f(x) = y…$

$\frac{p(p-1)(p-2…}{3!}$

$f(x) = 1 + …$

$x(x-…$

$\Rightarrow 1 - x …$

$= 1 - x + …$

Q) Consider a following table x and y

x	0	1	2	3
y	1	0	1	10

find the cubic equation by using Newtons forward Interpolation formula and also find $f(2.5)$.

(Difference table)

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	0	1 y_0	-1 Δy_0	$2\ \Delta^2 y_0$	$6\ \Delta^3 y_0$
x_1	1	0 y_1	1 Δy_1	$8\ \Delta^2 y_1$	
x_2	2	1 y_2	9 Δy_2		
x_3	3	10 y_3			

$$P = \frac{x_n - x_0}{h} = \frac{x_n - 0}{1} = \frac{x}{1} = P \quad \boxed{P = x}$$

$$f(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^2 y_0.$$

$$f(x) = 1 + x(-1) + \frac{x(x-1)}{\cancel{2}}(\cancel{2}) + \frac{x(x-1)(x-2)}{\cancel{3}\times\cancel{2}} \times \cancel{6}.$$

$$\Rightarrow 1 - x + x(x-1) + x(x-1)(x-2)$$

$$= 1 - x + x^2 - x + x^3 - 3x^2 + 2x$$

$$\boxed{f(x) = x^3 - 2x^2 + 1}$$

$$f(2.5) = (2.5)^3 - 2(2.5)^2 + 1$$
$$= 15.625 - 12.5 + 1$$
$$= 15.625 - 11.5$$
$$f(2.5) = \boxed{4.125}$$

---

if $x = 0.5$.

Without equation :- (direct Value) :-

$x = 0.5 \quad x_0 = 0 \quad h = 1$

$$p = \frac{x - x_0}{h} = \frac{0.5 - 0}{1} = 0.5$$

$$\boxed{p = 0.5}$$

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^2 y_3$$

$$f(x) = 1 + 0.5(-1) + \frac{(0.5)(-.5)}{\cancel{2}} \times \cancel{2} + \frac{0.5(-.5)(-1.5)}{\cancel{6}} \times \cancel{6}$$

$$f(x) = 1 - 0.5 - .25 + .375$$

$$f(x) = 0.625. \text{ Ans}$$

$x^3 - 2x^2 + 1$

$\Rightarrow 0.05 - 2\times(0.5)^2 + 1$

$\Rightarrow \cancel{0.05}\ 0.125 - 0.5 + 1 \Rightarrow \boxed{0.625}$

Q) In an examination the no. of Students who obtain the marks b/w certain limit there as follows:-

Marks.	30-40	40-50	50-60	60-70
No. of Students	52	36	21	14

= 0.5

find:-

(i) total no. of Students who obtain the marks below 35.

(ii) ~~total~~ no. of Students who obtain the marks above 65.

(iii) " " " " " " the marks b/w 35 & 45.

Calulate by using Netwon's forward Interpolation formula.

Sol)

~~x | y | Δy~~

~~upto 35~~
~~45~~
~~55~~
~~65~~

$$\begin{array}{r} 52 \\ +36 \\ \hline 88 \end{array} \qquad \begin{array}{r} 88 \\ +21 \\ \hline 109 \end{array} \qquad \begin{array}{r} 109 \\ +14 \\ \hline 123 \end{array}$$

Marks	marks upto x.	y	Δy	$\Delta^2 y$	$\Delta^3 y$
30-40	40	52.			
40-50	50	88	36		
50-60	60	109	21	-15	8
60-70	70	123	14	-7	

(i) $x = 35$ $\qquad p = \frac{x - x_0}{h} = \frac{35-40}{10} = -0.5$

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$f(x) = 52 + (-0.5)(36) + \frac{(-0.5)(-0.5-1)}{2} \times (-15)$$

$$+ \frac{(-0.5)(-0.5-1)(-0.5-2)}{6} \times 8.$$

$$\boxed{f(x) = 25.875}$$

$f(35) = 26$ // upto 35

Total no.

35 is

// Above

Total no

// b/w 45

∴

(ii) $x =$

$f(x)$

$f(x) =$

⇒ 52 +

⇒ 70

⇒

Total no. of Students upto 35 or below 35 is 26.

// Above 35.

Total no. of Students − below 35.

$$123 - 35 \Rightarrow \boxed{97}$$

// b/w 45 & 35.

$$\therefore \quad 52 - 26 \Rightarrow \boxed{26}$$

(ii) $x = 45$.

$$p = \frac{45-40}{10} = \frac{5}{10} = \frac{1}{2} = \boxed{0.5}$$

$$f(x) = 52 + (0.5)(36) + \frac{(0.5)(0.5-1)}{2} \times (-15) + \frac{(0.5)(0.5-1)(0.5-2)}{6} \times 8$$

$$\Rightarrow 52 + 18 + \frac{3.75}{2} + \frac{(-1.125)}{6} \times 8$$

$$\Rightarrow 70 + 1.875 - 1.5$$

$$\Rightarrow \boxed{72}.$$

// above 45

123 − 72. = $\boxed{51}$.

// b/w 35 & 45.

72 − 26 = $\boxed{46.}$.